Title: Move It

Brief Overview:

This unit is a means of studying linear data and functions, and working with linear data gathered from an experiment using a motion detector and a toy vehicle with constant velocity. Students will graph data points and find a linear equation which fits the data, both manually and using the TI Graphing Calculator.

Links to NCTM Standards:

• Mathematics as Problem Solving

Students will use the TI-82/83 and Calculator Based Ranger (CBR) or Calculator Based Laboratory (CBL) to collect and graph data.

• Mathematics as Communication

Students will use correct mathematical language to describe the results and significance of the experimental data and other data sets. Students will write about mathematical findings.

• Mathematics as Reasoning

Students will interpret and analyze graphs of linear functions. They will make conjectures about linear data. They will analyze variations of experimental data.

• Mathematical Connections

Students will recognize the connection between the graphical and the algebraic representations of a linear function. Students will use real phenomena to expand their knowledge of linear functions and will use that knowledge to interpret real-life situations. Students will be exposed to experimentation in the math classroom.

• Algebra

Students will use data to find the slope and the equation of a line.

• Functions

Students will work with linear functions.

Statistics

Students will collect and analyze data. Students will generate the line of best fit by calculator, and will use this line to make predictions from data.

Links to Maryland High School Mathematics Core Learning Goals:

• 1.1.1

The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

• 1.1.2

The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

• 1.2.1

The student will determine the equation for a line, solve linear equations, and describe the solutions using numbers, symbols, and graphs.

• 3.1.1

The students will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

• 3.2.1

The student will make informed decisions and predictions based upon the results of simulations and data from research.

• 3.2.2

The student will make predictions by finding and using a line of best fit and by using a given curve of best fit.

Grade/Level:

Grades 8-12; Algebra I, Algebra II, Pre-Calculus

Duration/Length:

Two to three class periods (variable)

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Graphing on the xy-coordinate plane
- Working with basic concepts of linear functions, including slope and finding equation of line given two points
- Using the TI-82/83 to enter data and to graph equations of lines

Objectives:

Students will:

- find the slope and equation of a line algebraically, given two points.
- use the statistical capabilities of the graphing calculator to enter data into lists, generate a graph, and find a line of regression.
- interpret a graph and predict values not obtained experimentally by using the equation of the line of best fit.
- associate linear data with an equation of a line.

Materials/Resources/Printed Materials:

- 1 CBR unit (Ranger) or CBL unit
- TI-82/TI-83 overhead projector
- TI-82/83 Graphing Calculator(s)
- Toy vehicle or animal which moves at constant velocity, battery-powered or wind-up.
- Student Activity Sheets #1 and #2, Student Assessment

Development/Procedures:

The teacher will use the CBR (or CBL with the Motion program) and a toy vehicle or animal which moves at a constant rate. The teacher should have a student help propel the vehicle in a straight line while using the CBR or CBL to measure the distance from a starting point. The data collected should be displayed on an overhead projector and also transferred to student calculators.

The teacher will guide the class to understand the graph displayed. Concepts to be discussed include independent and dependent variables, domain and range, and the linearity of the data. Students will then use Student Activity Sheet #1 to briefly review algebraic skills with linear functions, and complete it to further explore concepts and properties of linear graphs and functions.

Students will explore additional sets of linear data in Student Activity Sheet #2, to be completed the following class period. An Assessment is provided to conclude the activity.

Extension/Follow Up:

- Students can learn to use the coefficient of correlation or the residuals of a line of regression to judge whether the calculator-derived linear equation actually fits the data best.
- Students can study direct and indirect variation as a continuation of linear equations.
- Students can explore the concepts of velocity and acceleration, by repeating the experiment with a vehicle with non-constant velocity.
- Students can work with non-linear data and explore other mathematical functions.

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References:

<u>Texas Instruments Getting Started with CBR TM Including 5 Student Activities</u>, Texas Instruments Incorporated.

Teacher Notes: Experiment Set Up - "Move It"

Purpose: Students will be able to explore real-life linear relationships using distance and time.

Previous experience:

- Teachers will need a CBR with the RANGER program to run this experiment . If a CBR is not available, a motion sensor and a CBL unit with the MOTION program can be substituted.
- Teachers should practice the experiment before demonstrating in front of the class.

Materials needed:

- CBR
- TI-82/TI-83 Graphing Calculator
- CBR to Calculator cable
- Battery operated vehicle/toy car (preferably with on/off switch) or wind-up toy
- At least five link cords
- TI View Screen
- Classroom set of "Student Activity #1".

*Note to teacher - Make sure the toy car travels at a constant speed. Usually, toy cars made for younger children are better for this experiment. Additionally, make sure that toy car has a good reflective surface in order to obtain good data points. An index card can be taped to the back of the car to insure good data collection. Have students complete Pre Experiment on Student Activity #1 before proceeding with experiment. (Calculator instructions are for the TI-82 calculator.)

Experiment Procedure:

- 1. Place the CBI on the floor with the motion sensor facing the toy car.
- 2. Place the toy car at least .5 meters (1.5 feet) on the floor in front of the CBI. (False readings might be collected if the car is too close to the CBI). Make sure the car is facing away from the CBI.
- 3. Use cable to connect the CBI and calculator.
- 4. Run the Ranger program in calculator.
- 5. From the MAIN MENU choose 1: SETUP / SAMPLE.



6. For this experiment use the following settings:

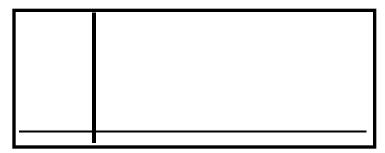
MAIN MENU START NOW

REALTIME: NO
TIME(S): 5
DISPLAY: DIST
BEGIN ON: CENTER]
SHOOTHING: LIGHT
UNITS: METERS

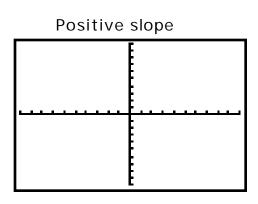
- 7. After you have changed the settings, scroll up to START NOW, press ENTER.
- 8. Follow directions on your calculator screen. When ready to start experiment, press enter. Make sure the path between the car and CBI remains clear. The Car should travel in a straight line.
- 9. Calculator will read TRANSFERRING... as it is getting the data from the CBI. A plot of the data should appear after the transfer is complete.
- 10. If the data does not look like a good sample, press ENTER. PLOT MENU will appear. Press 5: REPEAT SAMPLE and continue from step number 9 (above). Continue this process until you have obtained a good sample.
- 11. Once you have obtained a good sample, discuss the properties of the graph. Topics should include slope, y intercept, independent and dependent variables, and units of measurement (scale).
- 12. Use the links to pass the data in L₁ and L₂ to all students. See page 11 in the <u>Texas Instruments Getting Started with CBRTM</u> (handbook provided with the CBI) for directions on linking data to students.
- 13. Proceed with the students on the "Student Activity #1". If students are not able to finish, assign the reminder for homework.

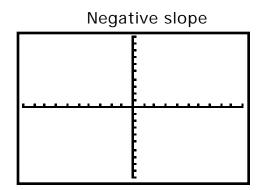
Student Activity #1 - Graphing Analysis

Pre Experiment - The experiment will feature a vehicle moving away from the motion detector. SKETCH a graph of your anticipated results of this experiment comparing time with distance.



- 1. Find the slope of the line between the points A and B.
 - a) A(2,4), B(4,7)
- b) A(-3,1), B(6,-2) c) A(.5,4), B(.75,-1)
- 2. Sketch a line with a positive slope, and one with a negative slope.





- 3. Find the equation of the line through the points C and D. Place your answer in y = mix + b form.
 - a) C(3,7), D(5,11)
- b) C(-1,4), D(1, -12)
- 4. For a line in y = mix + b form what does b represent on the graph?

You must have the data from the experiment before you continue this activity.

5. On your graphing calculator graph the data collected in the class experiment. Refer to Student Activity 4b for the data graphing procedure. Use Zoom, 9: Zoom State to create a good viewing screen. Sketch below the graph displayed on your viewing screen. Label your axes and indicate scaling. 6. Examine your graph. As time increases what happens to the distance the vehicle travels? 7. Describe the shape of your sketch. 8. In the graph, identify the independent variable and the dependent variable. Independent variable _____ Dependent variable _____ 9. Use Trace to locate and record two ordered pairs from the graph. (Round to the nearest hundredth- for example 3.06) (______ , _____) (_____ , ____) 10. Using the two points chosen in item #9 calculate the slope of the line between the two points and the equation of the line between the two points. (use y = mx + b form) slope _____ equation of the line ____

11. Place the equation from #10 in Y₁ on your calculator. Graph Y₁.

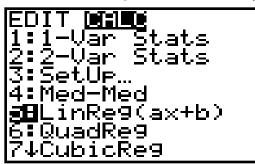
How does this line relate to the experiment?

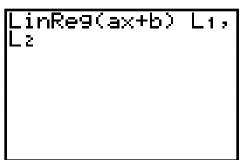
12.	Is the slope of your line positive or negative? How does the sign of the slope relate to the experiment.
	How does the slope of the line relate to the motion of the vehicle in the experiment ?
13.	Use your equation from #10 to predict the position of the test vehicle at time $t = 12$ seconds.
14.	Describe a situation, during the experiment, which might make your prediction unreasonable.
15.	Other classmates have chosen different points to complete the exercises above. How would you expect their answers to compare to yours ?
allo	e - In this lesson you have taken data and tried to find a line that models the data. The line ws you to predict the behavior of a particular situation. One process which is used to fit data to s or curves is called regression . In the next class, we will find regression equations on the ulator.
	The data from this activity will also be used in our next activity

Name	Date
1 141110	Date

Student Activity #2 - Regression and Best Fit Line

- 1. In the previous activity we used data from an experiment to create equations of lines. These lines can be used to predict other data points. Every student did not get the same equation to model the experiment. Some lines worked better than others. Mathematicians try to find the line which is provides the "best fit" for the data. One process to create such a line is known as regression.
- 2. The process
 - a) Make sure the experimental data from the "Move It" experiment is located in lists L₁ and L₂.
 - b) Press STAT ,CALC ,5: LinReg(ax+b)
 - c) After LinReg(ax+b) insert L₁, L₂ then ENTER (Note comma)

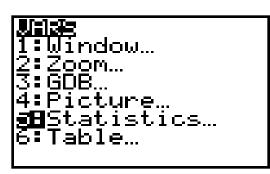


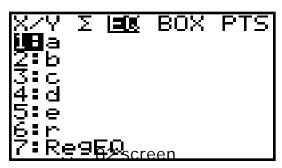


- d) The calculator now produces the Line of Best Fit.
- e) The Line of Best Fit is created using a mathematical algorithm.
- f) NOTE the calculator uses the letter a for the slope of the line.

3. How does the slope and y intercept of this regression line compare to the answers you got in Activity 1, question # 10?

- 4. Now we will graph the regression line and the original information on the same calculator screen.
 - a) To graph the regression line Go to Y₁ = then go to VARS, 5:STATISTICS,EQ,7:RegEQ then ENTER.
 - the regression equation now appears in Y_1





Note: Use 1:RegEQ on the TI-83

TI - 82 Screen

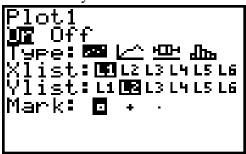
b) **To graph the data from the experiment** - Go 2nd STAT PLOT, Plot 1, Select On

Type: is asking for the type of graph you want. Choose the first diagram which creates a <u>scatter plot</u> of the data. ENTER

For XIIst choose L_1 (these are your x values)ENTER

For Ylist choose L_2 (these are your y values)ENTER

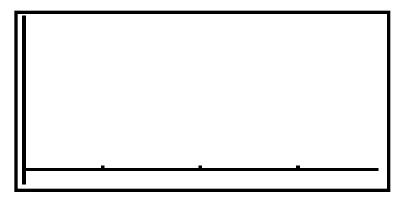
Mark: asks for the kind of symbol used to mark each data point. ENTER



- c) Pressing GRAPH will now produce both graphs. Try ZOOM STAT to get a good graphing window.
- 5. How do the graphs of the regression line and the plot of the experimental data compare?

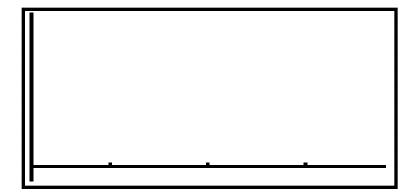
- 6. John and Cindy and their parents are traveling from Baltimore to Chicago. The most boring part of the trip is the Ohio Turnpike which is 240 miles long, with a marker to locate each of the 240 miles. At noon they enter the turnpike. At 1:30 they have completed 70 miles, at 2:00 they are at the 110 mile marker, at 3:00 PM the 180 mile marker and at 3:30 the 199 marker.
 - a) Create a table for the data presented in this problem. How will you record the time values ? (HINT How will you represent the time ?)

b) Create a scatter plot by hand for the data presented. Scale the axes so that the data will be clearly presented.



- c) Place the data points in L₁ and L₂.
- d) Using your calculator, compute the regression line and graph it along with your scatter plot.

 Equation of the regression line is ______
- e) Copy the scatter plot and the regression line below.



f) Using the regression equation predict the distance traveled at 3:48 PM.

g)	At what time would you predict John and Cindy leaving the turnpike ?
	Describe the method you used to answer this question?
h)	Do all the data points fall exactly on the regression line? Why or why not?
i)	John and Cindy's dad claims to always drive at 60 mph. What speed is the regression line indicating for the car's speed? Do these numbers agree? If not, why not?
j)	What is the y intercept of your regression line? What would you expect it to be based on the facts of the problem?
k)	In sketching the regression line for this problem should your regression line be drawn into the 2nd and/or 3rd quadrants . Why or why not ?

Teacher's Guide

Introduction This assessment is intended to follow the "Move It" learning unit. There are two parts to the assessment that can be combined in a single assessment or may be administered independently. Part 1 consists of selected response and constructed response questions. Part 2 is a performance activity. Part 2 may be given as a take-home assignment.

Objectives Covered This assessment assumes your students have received instruction covering the following objectives:

- Students will find the slope and equation of a line algebraically, given two points.
- Students will use the statistical capabilities of the graphing calculator to enter data into lists, generate a graph and find the equation of the Line of Best Fit.
- Student will interpret a graph and predict values not obtained experimentally by using the equation of the Line of Best Fit.
- Students will associate linear data with an equation of a line.

This task is to be done as an independent activity. You will be reading a set of general instructions to the students and then allowing them to work individually at their own pace. Make necessary accommodations for students with special needs.

Tools/Materials Needed for Assessment: pencil, copy of the assessment, and a TI-82/83 Graphing Calculator

Administering the Assessment

Part 1 of the assessment should take approximately 20 minutes to administer. Part 2 should take approximately 20 minutes to administer.

Distribute a copy of the assessment to each student.

SAY: Today you are going to use your mathematics skills to complete some items and activities in this assessment. You will complete everything on your own. Take time now to skim through the different parts of the assessment.

If you are administering both part of the assessment at the same time

SAY: You will complete Part 1 and then start Part 2.

Pause to allow students to look through the assessment. Have students write their name on the first page.

SAY: Are there any questions? You may begin.

When students have completed the assessment, collect their materials.

Mathematics Assessment - Part One - MOVE IT! Student Response Sheet

Date _____

For questio	ns 1 - 3 circl	e the correct	answer.		For questions 1 - 3 circle the correct answer.				
1. The slope	of the line that	goes through the	e two points (3,-1) (6, -3) is	: :					
a) 2/3	b) -3/2	c) 4/3	d) -2/3						
2. Which line	has a positive	slope?							
a)	b)		c)	d)					

USE THE FOLLOWING TABLE FOR QUESTIONS 3 AND 4.

x:	-4	-2	0	2
y:	16	10	4	-2

- 3. Given the data above the y-intercept is:
- a) 16 b) 10 c) 4 d) -2

- 4. Use algebra to find the linear equation of the data above. SHOW ALL WORK.

For questions 5 - 10 use the following table:

A McBurger Meals Truck runs lengthwise along a table at McBurgers. The following table shows the time and distance the truck traveled.

Time (seconds)	Distance (cm)
1	16
3	29
4	36
8	62

5.	Identify the independent variable:		
5.	Identify the dependent variable:		
7.	According to the table, as time increases, what is happening to the distance	e?	
8.	Using the table, put time into L_1 and distance into L_2 of your calculator. Regression to find the equation of the Line of Best Fit. Round to the new		
	What is the equation of the Line of Best Fit?		
9.	Predict the distance the McBurger truck has traveled at seven seconds		
10	. The table is 100 centimeters long. How many seconds until the truck fall may find your answer numerically, graphically, or algebraically.		

Mathematics Assessment - Part Two - MOVE IT! Student Response Sheet

Name	Date
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Read the following passage and answer the questions that follow.

Trisha is training to run in the Summer Fun 5K on June 1st. She starts training for the race on March 15th. On her first day of training she is able to run 5 kilometers in 38.18 minutes. At the end of the first week of training it takes her 36.98 minutes to run 5 kilometers. The table below shows Trisha's times at the end of each week for the first six weeks of training.

Week of	Time
Training	(minutes)
0	38.18
1	36.98
2	35.62
3	34.39
4	33.03
5	31.81
6	30.51

- 1. Plot the results of Trisha's weeks of training on your paper. Label your axes and indicate scale.
- 2. Explain why it would make sense to find the Line of Best Fit for this data.
- 3. Use your calculator to find the regression equation.
- 4. What does the y-intercept of your regression equation tell you about Trisha's training?
- 5. The Summer Fun 5K takes place at the end of week 10 of Trisha's training. Last year she completed the race in 22.32 minutes. If her 5K times continue to improve at the same rate that they did for the first six weeks, will she beat her previous year's time? Show evidence to support your answer.
- 6. According to the model, what time should Trisha have at the end of 25 weeks of training? Is it reasonable to assume that Trisha's times will continue to improve at the same rate? Explain your reasoning.

Scoring Guide: Move It Assessment - Part One

Mathematics Assessment - Part One - MOVE IT! Student Response Sheet

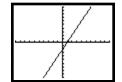
Name	Date

For questions 1 - 3 circle the correct answer.

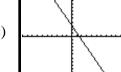
- 1. The slope of the line that goes through the two points (3,-1) (6, -3) is:
- a) 2/3
- b) -3/2
- c) 4/3
- d) -2/3
- 2. Which line has a positive slope?

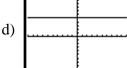


b)



c)





USE THE FOLLOWING TABLE FOR QUESTIONS 3 AND 4.

x:	-4	-2	0	2
y :	16	10	4	-2

- 3. Given the data above the y-intercept is:
- a) 16
- b) 10
- c) 4
- d) -2
- 4. Use algebra to find the linear equation of the data above. SHOW ALL WORK.

answer: y = -3x + 4

For questions 5 - 10 use the following table:

A McBurger Meals Truck runs lengthwise along a table at McBurgers. The following table shows the time and distance the truck traveled.

Time	Distance
(seconds)	(cm)
1	16
3	29
4	36
8	62

5. Identify the independent variable:

7. According to the table, as time increases, what is happening to the distance?

The distance increases.

8. Using the table, put time into L_1 and distance into L_2 of your calculator. Use Linear Regression to find the equation of the Line of Best Fit. Round to the nearest hundredth.

What is the equation of the Line of Best Fit? y = 6.58x + 9.44

- 9. Predict the distance the McBurger truck has traveled at seven seconds. <u>55.5 cm</u>
 *Teachers will need to use the students model given in answer #8 to find this answer.
- 10. The table is 100 centimeters long. How many seconds until the truck falls off the table? You may find your answer numerically, graphically, or algebraically.

 14 seconds

 *Teachers will need to use the students model given in answer #8 to find this answer.

Scoring Guide: Rubric for "Move It" Assessment - Part Two

- **3:** This student demonstrates a clear ability to correctly graph data. The student correctly uses the graphing calculator to find the equation of a line of regression. The student is able to correctly interpret, analyze and predict using a linear model. The student demonstrates a complete understanding that linear data can be modeled by the equation of a line.
- 2: This student demonstrates some ability to correctly graph data. The student may or may not use the graphing calculator correctly to find the equation of a line of regression. The student makes an attempt to correctly interpret, analyze and predict using a linear model. The student demonstrates some understanding that linear data can be modeled by the equation of a line.
- 1: This student demonstrates little ability to correctly graph data. The student may or may not use the graphing calculator correctly to find the equation of a line of regression. The student makes an attempt to interpret, analyze and predict using a linear model. The student demonstrates little understanding that linear data can be modeled by the equation of a line.
- **0:** This student demonstrates little or no ability to correctly graph data. The student does not use the graphing calculator correctly to find the equation of a line of regression. The student cannot correctly interpret, analyze or predict using a linear model. The student does not demonstrate understanding that linear data can be modeled by the equation of a line.